Pg.132 Solve the following

i. UT × 3 = PUT

50 × 3 = 150 U=5, T=0, P=1

v. PP × QQ = PRP

22 × 11 = 242 P=2, Q=1, R=4

Pg.132 Figure it out

1. If 31z5 is a multiple of 9, where z is a digit, what is the value of z? Explain why there are two answers to this problem.

Soln:

Digit sum = 3 + 1 + 5 + z

= 9 + z

9 + z should be a multiple of 9.

So, z can be either 0 or 9.

If z=0, 9+z = 9+0 = 9 (divisible by 9)

If z=9, 9+z = 9+9 = 18 (divisible by 9)

Note: Use the three dots sign shown here, ∴ wherever needed in the place of Therefore.

Note: the sign, '=>' written in the notes is to be read as, 'that implies'. Write this in your note as arrow sign drawn near this note. ⇒

2. "I take a number that leaves a remainder of 8 when divided by 12. I take another number which is 4 short of a multiple of 12. Their sum will always be a multiple of 8", claims Snehal. Examine his claim and justify your conclusion."I take a number that leaves a remainder of 8 when divided by 12. I take another number which is 4 short of a multiple of 12. Their sum will always be a multiple of 8", claims Snehal. Examine his claim and justify your conclusion.

Padlet

Soln:

12a+8, 12b-4

Sum = 12a+8+12b-4 = 12(a+b)+8-4 = 12(a+b)+4

a=1, b=2

12(1+2)+4 = 12(3)+4 = 36+4 = 40 (divisible by 8)

a=0, b=2

12(0+2)+4 = 12(2)+4 = 24+4 = 28 (not divisible by 8)

Therefore, Snehal's claim is wrong.

Therefore, The sum of a number that leaves a remainder of 8 when divided by 12 and a number which is 4 short of a multiple of 12, will not always be a multiple of 8.

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3. When is the sum of two multiples of 3, a multiple of 6 and when is it not? Explain the different possible cases, and generalise the pattern.

Soln:

Let 3a, 3b be the multiples of 3.

Sum = 3a+3b = 3(a+b)

=> 3(a+b) is definitely divisible by 3

=> It should be also divisible by 2, to be divisible by 6

If a+b = Even number, the product will be divisible by 2

3 (odd) x (a+b) (even) Odd x Even = Even number

Padlet

Then it is divisible by 2. Therefore, it is divisible by 6.
If a+b = Odd number, then 3 (Odd) x (a+b) (Odd) Odd x Odd = Odd number
Then it is not divisible by 2. Therefore, it is not divisible by 6.

5. If 48a23b is a multiple of 18, list all possible pairs of values for a and b.

=> 48a23b is a multiple of 2 and 9
Digit sum => 4+8+a+2+3+b = 17+a+b
17+1=18 which is divisible by 9, 2
Therefore, a+b=1 => a=1, b=0
Therefore, as 481230 is a multiple of both 9 and 2, it is a multiple of 18

6. If 3p7q8 is divisible by 44, list all possible pairs of values for p and q.

=> 3p7q8 is divisible by 4, 11
Divisibility of 4 => Last two digits should be divisible by 4
$\frac{q8}{4}$ q=0 : 8, q=1 : 18, q=2 : 28, q=3 : 38, q=4 : 48, ...
q should be an even number. Therefore, q = 0,2,4,6,8,...
Divisibility of 11 => Sum of alternate digits should be 0 or a multiple of 11
3p7q8 3+7+8 = 18
18-(p+q) = 11 => 18-7 = 11 Therefore, p+q=7
So, if q=0, p+0=7 => 7+0=7 (q is always an even number) (1) p=7, q=0

Padlet

If q=2, p+2=7 => p=7-2 => p=5 (2) p=5, q=2
If q=4, p+4=7 => p=7-4 => p=3 (3) p=3, q=4
If q=6, p+6=7 => p=7-6 => p=1 (4) p=1, q=6
Therefore the possible pairs are (7,0), (5,2), (3,4), (1,6)

7. Find three consecutive numbers such that the first number is a multiple of 2, the second number is a multiple of 3, and the third number is a multiple of 4. Are there more such numbers? How often do they occur?

Soln:

2,3,4
4,5,6
6,7,8
8,9,10
10,11,12
12,13,14
14,15,16

14 - 2 = 12, 15 - 3 = 12, 16 - 4 = 12

Therefore, it occurs after every 12 numbers. Because, the LCM of 2,3,4 = 12. Yes, there are infinite numbers like this.

10. Write a 6-digit number that it is divisible by 15, such that when the digits are reversed, it is divisible by 6.

Soln:

A number is divisible by 5, then it is divisible by 3 and 5.

Therefore, the last digit of the number be 5
The reverse of the number is divisible by 6. So, it should be divisible by 2 and 3.
Therefore, the first digit should be an even number, as it becomes the last digit when reversed and the number be 2abcd5 (Sum of digits must be divisible by 3)
Digit Sum = 2+a+b+c+d+5 = 7+a+b+c+d => 7+2=9
Therefore, a+b+c+d=2 => 210015 satisfies the given condition.

12. Determine whether the statements below are 'Always True', 'Sometimes True', or 'Never True'. Explain your reasoning.

(i) The product of a multiple of 6 and a multiple of 3 is a multiple of 9.

6a x 3b = 18ab = 9(2ab) => multiple of 9. Therefore, the given statement is always true.

(ii) The sum of three consecutive even numbers will be divisible by 6.

2n+(2n+2)+(2n+4) = 2n+2n+2+2n+4 = 6n+6 = 6(n+1) => divisible by 6

Therefore, the given statement is always true.

(iii) If abcdef is a multiple of 6, then badcef will be a multiple of 6.

In badcef, the last digit is even, digit sum is a multiple of 3. Therefore, badcef is a multiple of 6. Therefore, the given statement is always true.

(iv) 8 (7b – 3) – 4 (11b + 1) is a multiple of 12.

56b-24-44b-4 = 56b-44b-24-4 = 12b-24-4 = 12(b-2)-4.

Let b=3, => 12(3-2)-4 = 12(1)-4 = 12-4 = 8 => not divisible by 12.

Therefore, it is not a multiple of 12 which says the given statement is never true.

13. Choose any 3 numbers. When is their sum divisible by 3? Explore all possible cases and generalise.

i. 3,6,9 => 3+6+9=18 is divisible by 3 and Remainder = 0,0,0, Sum of remainders = 0
ii. 4,7,10 => 4+7+10=21 is divisible by 3 and Remainder = 1,1,1, Sum of remainders = 3
iii. 4,5,6 => 4+5+6=15 is divisible by 3 and Remainder = 1,2,0, Sum of remainders = 3
iv. 7,8,9 => 7+8+9=24 is divisible by 3 and Remainder = 1,2,0, Sum of remainders = 3
ii. 5,8,11 => 5+8+11=24 is divisible by 3 and Remainder = 2,2,2, Sum of remainders = 6
Therefore, Sum of three numbers is divisible by 3 only if sum of the remainders of each number is divisible by 3.

15. Solve the cryptarithms — (i) EF × E = GGG (ii) WOW × 5 = MEOW

i. EF × E = GGG : 37 × 3 = 111 ; E=3, F=7, G=1

ii. WOW × 5 = MEOW : 575 × 5 = 2875 ; W=5, O=7, E=8, M=2

Divisibility rules:

2: Last digit is even number or 0; 3: Digit sum is a multiple of 3; 10: Last digit 0;
4: Last two digits are divisible by 4; 5: Last digit is 0 or 5; 6: Divisible by both 2 and 3;
8: Last three digits are divisible by 8; 9: Digit sum = 0 or a multiple of 9;
11: Difference of alternate digits = 0 or a multiple of 11; 7: Double its last digit and subtract that value from the rest of the number, the number you got should divided by 7